LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Wintersemester 2018/19

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Algebraic Geometry

Sheet 2

Unless specified otherwise, we will always work over an algebraically closed field k.

Exercise 1. (4 points) *Closed, open or dense?*
Let $k = \mathbb{C}$ and decide for each of the following subsets $S \subset \mathbb{A}^2_{\mathbb{C}}$ whether they are closed, open or dense (in the Zariski topology):

(a) $S = \{(t, t^2) \mid t \in \mathbb{C}\}$;

(b) $S = \{(t, st) \mid s, t \in \mathbb{C}\}$;

(c) $S = \{(s, t) \mid s, t \in \mathbb{Q}\}$;

(d) $S = \{(t, 2^t) \mid t \in \mathbb{Z}\}$.

Exercise 2. (4 points) *Irreducible components of an algebraic set.*
Decompose the algebraic set

$$X := V(x_1^2 - x_2x_3,\ x_1x_3 - x_1) \subset \mathbb{A}^3_k$$

into its irreducible components.

Exercise 3. (4 points) *The quadric cone.*
Let $X := V(x_1x_2 - x_3x_4) \subset \mathbb{A}^4$.

(a) Show that $k[X]$ is not a unique factorization domain;

(b) Find a regular function $f \in k[X]$ whose vanishing locus $V_X(f) \subset X$ has the property that none of its irreducible components are given by the vanishing of a single regular function on X.

Exercise 4. (4 points) *A criterion for a regular map to be an isomorphism.*
Let $\phi : X \to Y$ be a regular map between affine algebraic sets. Show that ϕ is an isomorphism if $\phi^* : k[Y] \to k[X]$ is an isomorphism.
(**Hint:** Let $X \subset \mathbb{A}^n$ with affine coordinates $t_1, \ldots, t_n$ on $\mathbb{A}^n$. To guess the correct formula for the inverse ψ of ϕ, assume for a moment that it exists. Then $\psi = (\psi_1, \ldots, \psi_n)$ with regular functions $\psi_i \in k[Y]$ which are explicitly given by $\psi_i = \psi^*(t_i)$, where ψ^* is an inverse of ϕ^*.)

Hand in: before noon on Monday, October 29th in the appropriate box on the 1st floor.