LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Sommersemester 2019

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Algebraic Geometry 2

Sheet 10

Exercise 1. (4 points) *Global regular functions of proper integral schemes over fields are constant.*
Let k be an algebraically closed field and let X be an integral scheme which is proper over k. Show that $\Gamma(X, \mathcal{O}_X) \cong k$.
(**Hint:** We have seen last semester that projective varieties have this property and you may use a similar idea of proof here.)

Exercise 2. (4 points) *Proper $\Rightarrow$ reduced?*
Let k be a field. Is every proper scheme over k reduced?

Exercise 3. (4 points) *When are Zariski open subsets proper?*
Let k be a field and let X be a proper integral scheme over k. Show that a non-empty Zariski open subset $U \subset X$ is proper over k if and only if $U = X$.

Exercise 4. (4 points) *Which affine schemes over a field are proper over k?*
Let k be a field. Show that a closed integral subscheme $X \subset \mathbb{A}^n_k$ is proper over k if and only if it is zero-dimensional.
(**Hint:** You may use Exercise 3.)

Hand in: before noon on Monday, July 8th in the appropriate box on the 1st floor.