LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Sommersemester 2019

Prof. Dr. Stefan Schreieder
Dr. Feng Hao

Algebraic Geometry 2

Sheet 7

Exercise 1. (4 points) *Locally of finite type*
Let $f : X \to Y$ be a morphism of schemes. Show that f is locally of finite type if and only if for any affine open subset $V = \operatorname{Spec} A \subset Y$, $f^{-1}(V)$ can be covered by affine open subsets $U_i = \operatorname{Spec} B_i$ for A-algebras B_i that are of finite type over A (i.e. finitely generated as A-algebras).

Exercise 2. (4 points) *Generically finite morphisms*
Let $f : X \to Y$ be a morphism between integral schemes. We say that f is dominant, if the image of f is dense in Y. We say that f is generically finite, if $f^{-1}(\eta)$ is finite for the generic point $\eta \in Y$.
Assume that f is of finite type, dominant and generically finite. Show that there is a non-empty open subset $U \subset Y$, such that $f^{-1}(U) \to U$ is a finite morphism of schemes (where $f^{-1}(U) \subset X$ carries the canonical open subscheme structure).
(**Hint:** Show first that the function field of X is a finite field extension of the function field of Y.)

Exercise 3. (4 points) *Closed points of finite type schemes are dense*
Let X be a scheme of finite type over a field. Show that the closed points of X are dense. Give an example to show that this does not need to be true if X is not of finite type.

Exercise 4. (4 points) *Properties of morphisms of finite type*
A morphism of schemes $f : X \to Y$ is quasi-compact if there is an open affine covering $Y = \bigcup V_i$ such that $f^{-1}(V_i)$ is quasi-compact for all i. You may use without proof that this is equivalent to asking that for any open affine subset $V \subset Y$, $f^{-1}(V)$ is quasi-compact.
Prove the following assertions:

(a) A closed immersion is a morphism of finite type.

(b) An open immersion which is quasi-compact is of finite type.

(c) The composition of morphisms of finite type is of finite type.

(d) If $f : X \to Y$ is quasi-compact and $g : Y \to Z$ is any morphism such that $g \circ f$ is of finite type, then f is of finite type.

Hand in: before noon on Monday, June 17th in the appropriate box on the 1st floor.