LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Sommersemester 2019

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Algebraic Geometry 2

Sheet 9

Exercise 1. (4 points) *Separated?*
Let k be a field. Consider the schemes X_1 and X_2 over k from Exercise 4 on sheet 4. Decide for $i = 1, 2$ whether X_i is separated.

Exercise 2. (4 points) *When are morphisms which coincide generically actually the same?*
Let S be a scheme and let X and Y be S-schemes. Let $f, g : X \to Y$ be two morphisms of S-schemes which agree on a dense open subset of X.

(a) Show that if X is reduced and Y is separated, then $f = g$.

(b) Give examples that show that both conditions in part (a) are necessary.

Exercise 3. (4 points) *Finite morphisms are proper*
Show that any finite morphism of schemes is proper.

Exercise 4. (4 points) *Corollary of valuative criterion of properness*
Prove the following statements, where all schemes are noetherian.

(a) A closed immersion is proper.

(b) Composition of proper morphisms is proper.

(c) Proper morphisms are stable under base change.

(d) Products of proper morphisms are proper.

(e) If $f : X \to Y$ and $g : Y \to Z$ are two morphisms with $g \circ f$ proper and g separated, then f is proper.

(f) Properness is local on the base.

Hint: Item (e) is proven in II.4.8 of Hartshorne's book.

Hand in: before noon on Monday, July 1st in the appropriate box on the 1st floor.