Sommersemester 2019

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# Algebraic Geometry 2

Sheet 3

**Exercise 1.** (4 points) *Standard open subsets are affine*
Let $A$ be a ring. Recall that the standard open subsets of $\operatorname{Spec} A$ are of the form $U_f = \operatorname{Spec} A \setminus V(f)$, where $f \in A$. Show that for any $f \in A$, the locally ringed spaces

$$(U_f, \mathcal{O}_{\operatorname{Spec} A}|_{U_f}) \quad \text{and} \quad (\operatorname{Spec} A_f, \mathcal{O}_{\operatorname{Spec} A_f})$$

are isomorphic to each other.

**Exercise 2.** (4 points) *Standard open subsets form a basis*

(a) Let $A$ be a ring. Show that the standard open subsets of $\operatorname{Spec} A$ form a basis of the topology of $\operatorname{Spec} A$.

(b) Conclude from the previous exercise that for any scheme $X$, for any point $p \in X$ and for any open neighbourhood $V \subset X$ of $p$, there is a neighbourhood $U \subset V$ of $p$ which is an affine scheme. That is, affine schemes form a basis for the topology of any scheme.

**Exercise 3.** (4 points) *When is* $\operatorname{Spec} A$ *Hausdorff?*
Let $A$ be a ring. Show that $\operatorname{Spec} A$ is Hausdorff if and only if any prime ideal of $A$ is maximal.

**Exercise 4.** (4 points) *Maps of locally ringed spaces.*

(a) Let $\varphi : k \to K$ be a homomorphism of fields. (Note that $\varphi$ is automatically injective and so $\varphi$ yields a field extension.) Let $X = \operatorname{Spec} K$ and $Y = \operatorname{Spec} k$. Describe the induced map $(X, \mathcal{O}_X) \to (Y, \mathcal{O}_Y)$ of locally ringed spaces.

(b) Let $A$ be a ring and let $I \subset A$ be an ideal. Describe the map of locally ringed spaces that is induced by the quotient map $\varphi : A \to A/I$.

(c) Let $A$ be a ring and let $S \subset A$ be a multiplicative subset. Describe the map of locally ringed spaces that is induced by the natural map $\varphi : A \to S^{-1}A$.

**Hand in:** before noon on Monday, May 13th in the appropriate box on the 1st floor.