LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Sommersemester 2019

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Algebraic Geometry 2

Sheet 1

This Exercise sheet contains short problems which are due on Monday noon of the second week of the summer term.

Exercise 1. (4 points) *Vanishing sets*

(a) Let k be an algebraically closed field and let X be a k-variety. For an ideal of regular functions $I \subset k[X]$, show that $V_X(I) \subset X$ corresponds to all maximal ideals in $k[X]$ which contain I.

(b) Consider the affine scheme $X := \operatorname{Spec} \mathbb{Z}$. Compute the vanishing sets of the following ideals

(i) $I = (0) \subset \mathbb{Z}$;

(ii) $I = (2, 3) \subset \mathbb{Z}$;

(iii) $I = (12) \subset \mathbb{Z}$.

Exercise 2. (4 points) *Functoriality works for prime ideals*
Let $\varphi : A \to B$ be a homomorphism of rings (commutative with unit as usual).
Show that for any prime ideal $\mathfrak{p} \subset B$, the preimage $\varphi^{-1}\mathfrak{p}$ is a prime ideal in A.

Exercise 3. (4 points) *Some simple examples of affine schemes*
Describe the following topological spaces, where k denotes an arbitrary field:

(a) $\operatorname{Spec} k$;

(b) $\operatorname{Spec} k[t]/t^2$;

(c) $\operatorname{Spec} k[t]$;

(d) $\operatorname{Spec} \mathbb{Z}/6$;

(e) $\operatorname{Spec} \mathbb{Z}/12$;

(f) $\operatorname{Spec} \mathbb{Z}$.

Hand in: before noon on Monday, April 29rd in the appropriate box on the 1st floor.