**Exercise 4**:
"⇒" Assume that $X = \operatorname{Spec} A$ is proper over $\operatorname{Spec} k$, we want to show that $\dim X = 0$.
*Proof:* First we recall a theorem from Hartshorne.
**Theorem A:** Let $A$ be a subring of a field $K$, then the integral closure of $A$ in $K$ is the intersection of all valuation rings of $K$ containing $A$.

Denote $K := \mathrm{Frac}(A)$, where $A$ is a domain, since $X$ is integral. Then consider any valuation of $K$ containing $k$, i.e., $k \subset R \subset K$, we have first by the above Theorem A, the integral closure of $k$ in $K$ is

$$\bar{k}^K = \bigcap_{k \subset R \subset K} R,$$

where $R$ are the valuation rings of $K$ containing $k$. Also, by the properness of $X$, for any valuation ring $R$ of $K$ we have the following diagram

$$\begin{tikzcd}
K & A \arrow[l, hook', "i"'] \arrow[dl, dashed, "h"] \\
R \arrow[u, hook, "j"] & k \arrow[l] \arrow[u]
\end{tikzcd}$$

and a unique homomorphism $h : A \to R$, with $h$ an injection, since both $i$ and $j$ injective homomorphisms. Hence especially we get

$$A \subset \bigcap_{k \subset R \subset K} R.$$

Thus by above argument we have $A$ is integral over $k$, which implies $\mathrm{Transdeg} K/k = 0$. Hence $\dim X = 0$.