Winter term 2023/24 9 January 2024

Topology I

Sheet 11

Exercise 1. Let B be a CW-complex and let $p\colon E \to B$ be a covering map. Show that the subspaces $\{p^{-1}(\mathrm{sk}_n(B))\}_{n\geq -1}$ give a CW-structure on E.

Exercise 2. Consider the space $X = S^2 \cup C$ where $C = \{(0,0,z) \in \mathbb{R}^3 \mid -1 \leq z \leq 1\}$ (see Exercise 4, Sheet 7). Describe all connected coverings of X and show that they are Galois.

Exercise 3. Show that, for all $n \geq 1$, the degree $\deg\colon [S^n, S^n] \to \mathbb{Z}$ is surjective. [Hint: Show that the map $S^1 \to S^1$, $z \mapsto z^n$ has degree n. Instead of using an isomorphism of $\pi_1(S^1)$ with $H_1(S^1)$, you may also use additivity of homology on wedge-sums: The inclusions $X_i \to X_1 \vee X_2$, $i = 1, 2$, induce isomorphisms $H_n(X_1) \oplus H_n(X_2) \cong H_n(X_1 \vee X_2)$ for all $n \geq 1$.]

Exercise 4. Prove the surjectivity theorem from the lecture:

(a) Let $n \geq 1$ and let $f\colon S^n \to S^n$ be a map of non-zero degree. Show that f is surjective.

(b) Let $n \geq 2$ and let $f\colon D^n \to D^n$ be a map which restricts to a map $f|_{\partial D^n}\colon S^{n-1} \to S^{n-1}$ of non-zero degree. Show that f must be surjective.

This sheet will be discussed in the week of 15 January 2024.