



LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
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MATHEMATISCHES INSTITUT



Winter term 2023/24

2 January 2024

Topology I

Sheet 10 (Trial Exam 90 mins)

Problem 1. Let there be given topological spaces E, E', B , and maps $E \xrightarrow{f} E' \xrightarrow{g} B$. Assume that B is locally path-connected.

- (a) Suppose that f and gf are covering maps and f is surjective. Show that g is a covering map.
- (b) Show that if f is not assumed surjective in (a), then g need not be a covering map.

Problem 2. Let (X, x) be a based space and G a group. Let there be given a group homomorphism $\varphi: \pi_1(X, x) \rightarrow G$. Construct a space Y and a map $f: X \rightarrow Y$ such that $\pi_1(Y, f(x)) \cong G$ and the map induced by f on fundamental groups $\pi_1(f): \pi_1(X, x) \rightarrow \pi_1(Y, f(x)) \cong G$ is equal to φ .

[Hint: Factorise φ as the composite $\pi_1(X, x) \rightarrow H \hookrightarrow G$ where H is the image of φ . Solve the problem separately for the surjection $\pi_1(X, x) \rightarrow H$ and the injection $H \hookrightarrow G$.]

Problem 3.

- (a) Let G be a group and let $H, K \subseteq G$ be subgroups. Show that G/H and G/K are isomorphic as G -sets if and only if there exists $g \in G$ such that $gHg^{-1} = K$.
- (b) Consider the space $X = \mathbb{RP}^3 \times \mathbb{RP}^3 \times L(3; 1, 1)$. Determine the number of 2-, 3-, and 5-sheeted connected coverings of X up to isomorphism.

This sheet will be discussed in the week of 8 January 2024.