LMU

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UNIVERSITÄT
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MATHEMATISCHES INSTITUT

Winter term 2023/24 12 December 2023

Topology I

Sheet 9

Exercise 1. For all $m, n \geq 1$ find a group G with order $|G| = mn$ acting on a space X such that the projection $X \to X/G$ is an n-sheeted covering. (A covering map $p\colon E \to B$ is called *n-sheeted* if, for all $b \in B$, $|\mathrm{Fib}_b| = n$.)

Exercise 2. Let G be a group which acts covering-like on spaces X and Y, and let $f\colon X \to Y$ be a G-equivariant map, i.e., a map such that $f(gx) = gf(x)$ for all $x \in X$ and $g \in G$. Let $p\colon X \to X/G$ and $p'\colon Y \to Y/G$ be the projections, and write $\bar{f}\colon X/G \to Y/G$ for the map induced by f. For $x \in X$ let $\mathfrak{m}_p\colon \pi_1(X/G, p(x)) \to G$ (respectively, $\mathfrak{m}_{p'}$) be the map constructed in Lemma 3.12 (where it was denoted q). Show that, for every $x \in X$, the diagram

$$\begin{array}{ccc} \pi_1(X/G, p(x)) & \xrightarrow{\bar{f}_*} & \pi_1(Y/G, \bar{f}(p(x))) \\ & {\scriptstyle \mathfrak{m}_p}\searrow \quad \swarrow{\scriptstyle \mathfrak{m}_{p'}} & \\ & G & \end{array}$$

commutes.

Exercise 3. Show that there are no covering maps

(i) $K \to T^2$

(ii) $\mathbb{R}P^n \to S^n$, $n \geq 2$.

Exercise 4. Show that a finite wedge of circles is covered by a tree, i.e., show that for all $n \in \mathbb{N}$ there is a tree T and a covering map $T \to \bigvee^n S^1$.

This sheet will be discussed in the week of 18 December 2023.