

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN



Winter term 2023/24

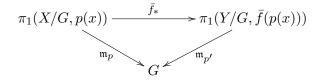
12 December 2023

Topology I

Sheet 9

Exercise 1. For all $m, n \ge 1$ find a group G with order |G| = mn acting on a space X such that the projection $X \to X/G$ is an *n*-sheeted covering. (A covering map $p: E \to B$ is called *n*-sheeted if, for all $b \in B$, $|Fib_b| = n$.)

Exercise 2. Let G be a group which acts covering-like on spaces X and Y, and let $f: X \to Y$ be a G-equivariant map, i.e., a map such that f(gx) = gf(x) for all $x \in X$ and $g \in G$. Let $p: X \to X/G$ and $p': Y \to Y/G$ be the projections, and write $\overline{f}: X/G \to Y/G$ for the map induced by f. For $x \in X$ let $\mathfrak{m}_p: \pi_1(X/G, p(x)) \to G$ (respectively, $\mathfrak{m}_{p'}$) be the map constructed in Lemma 3.12 (where it was denoted q). Show that, for every $x \in X$, the diagram



commutes.

Exercise 3. Show that there are no covering maps

- (i) $K \to T^2$
- (ii) $\mathbb{R}P^n \to S^n, n \ge 2.$

Exercise 4. Show that a finite wedge of circles is covered by a tree, i.e., show that for all $n \in \mathbb{N}$ there is a tree T and a covering map $T \to \bigvee^n S^1$.

This sheet will be discussed in the week of 18 December 2023.