MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
Prof. Dr. Peter Müller

Summer term 2014
Sheet 11
18.06.2014

Functional Analysis

E42 [6 points]. Let $Y \subsetneq X$ be a closed subspace of a normed space $(X, \|\cdot\|)$. Prove that for $x_0 \in X \backslash Y$ there exists $l \in X^*$ such that $\|l\| = 1$, $l|_Y = 0$ and $l(x_0) = \operatorname{dist}(x_0, Y)$.

[*Hint:* Define a suitable functional on $\operatorname{span}(Y \cup \{x_0\})$ and extend it to X.]

E43 [8 points]. Prove that $L^1(\mathbb{R}) \subsetneq (L^\infty(\mathbb{R}))^*$ in the sense of the canonical embedding.

[*Hint:* Consider the linear functional $l : \mathcal{L} \to \mathbb{K}, f \mapsto \lim_{x\to\infty} f(x)$, where $f \in \mathcal{L} :\Leftrightarrow$ there exists a representative $\tilde{f}$ of f, such that $\lim_{x\to\infty} \tilde{f}(x) =: \lim_{x\to\infty} f(x)$ exists. Why is this limit well-defined?]

E44 [5 points]. Let X and Y be Banach spaces. Prove that a bilinear map $\tau : X \times Y \to \mathbb{K}$ that is separately continuous (i.e. for each fixed $x \in X$ and $y \in Y$, we have $\tau(x, \cdot) \in Y^*$ and $\tau(\cdot, y) \in X^*$) is *jointly continuous*, i.e. if $x_n \to x$ and $y_n \to y$, then $\tau(x_n, y_n) \to \tau(x, y)$.

E45 [5 points]. Let $y = (y_k)_{k\in\mathbb{N}}$ be a sequence in $\mathbb{K}$. Show that if $\sum_{k=1}^\infty y_k x_k$ exists for all $x \in c_0$, then y belongs to ℓ^1.

Please hand in your solutions until next **Wednesday (25.06.2014)** *before* **12:00** *in the designated box on the first floor. Don't forget to put your name and the letter of your exercise group on all of the sheets you submit.*

For more details please visit http://www.math.lmu.de/~gottwald/14FA/