

**Conference “Motives: arithmetic, algebraic geometry and topology  
under the white-blue sky ”**

**Schedule**

	Monday, July 3	Tuesday, July 4	Wednesday, July 5
9:25	<i>Opening</i>		
9:30 - 10:30	Hélène Esnault	Detlev Hoffmann	Tom Bachmann
11:00 - 12:00	Bruno Klingler	Alexander Vishik	Denis-Charles Cisinski
12:00 - 14:00	<i>Lunch</i>	<i>Lunch</i>	
14:00 - 15:00	Claire Voisin	Nikita Karpenko	<i>Free afternoon</i>
15:30 - 16:30	Jörg Wildeshaus	Bruno Kahn	<i>Travel to Tegernsee</i>

	Thursday, July 6	Friday, July 7
9:30 - 10:30	Niko Naumann	Joseph Ayoub
11:00 - 12:00	Alexander Schmidt	Marc Levine
12:00 - 14:00	<i>Lunch</i>	
14:00 - 15:00	Nobuaki Yagita	
15:30 - 16:30	Kirill Zainoulline	
19:00	<i>Conference Dinner in Restaurant “Hunsinger” (registration required)</i>	

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München, July 3–7, 2017

**Joseph Ayoub (Zürich):** *On the conservativity conjecture*

The conservativity conjecture predicts that an algebraic correspondance between Chow motives is invertible if and only if its action on cohomology is invertible. I’ll describe parts of work in progress aiming at proving this conjecture in characteristic zero and for a classical Weil cohomology theory.

**Tom Bachmann (Essen):** *Multiplicative transfer in motivic homotopy theory*

(Joint with Marc Hoyois). If  $S' \rightarrow S$  is a finite étale morphism of schemes and  $X$  is a smooth quasi-projective  $S'$ -scheme, then the Weil transfer  $R_{S'/S}X$  is also a smooth quasi-projective scheme, which should be thought of as the “ $S'$ -indexed product of  $X$ ”. Following this intuition, one may define multiplicative pushforwards in (stable) motivic homotopy theory: symmetric monoidal functors  $N_{S'/S}: SH(S') \rightarrow SH(S)$ . Using these functors, one may define a motivic version of highly commutative ring spectra (which we call normed spectra) that is stronger than the usual notion of  $E_\infty$  Ring spectra.

In this talk we give an overview of the motivic norm functors and properties of motivic normed spectra; time permitting we will discuss various examples, the relationship with the Hill–Hopkins–Ravenel norms, slices of normed spectra, etc.

**Denis-Charles Cisinski (Regensburg):** *Homotopy K-theory of derived schemes*

We shall see how to understand homotopy K-theory of derived (or even spectral) schemes in terms of ordinary schemes. This is achieved by extending homotopy theory of schemes to derived geometry. As a concrete consequence, one can understand the cup product in our favourite cohomologies in terms of derived fibre product for possibly singular schemes, for instance. This is a report on joint work with Adeel A. Khan.

**Hélène Esnault (Berlin):** *Rigid connections,  $F$ -isocrystals and integrality*

On a smooth  $X$  is a smooth complex projective variety, Carlos Simpson conjectures that a rigid integrable connection is motivic. This in particular implies that the monodromy is integral. We prove the integrality conjecture when the connection defines a smooth moduli point. To this aim, we prove that the mod  $p$  reduction of a rigid integrable connection has the structure of an isocrystal with Frobenius structure. We also prove that rigid integrable flat connections with vanishing  $p$ -curvatures are unitary. This allows one to prove new cases of Grothendieck's  $p$ -curvature conjecture.

Joint work with Michael Groechenig.

**Detlev Hoffmann (Dortmund):** *Equivalence relations for quadratic forms*

We compare equivalence relations for quadratic forms over fields and their associated quadrics, such as (stable) birational equivalence of quadrics, their motivic equivalence or similarity of quadratic forms. We survey some of the known results and highlight some open problems.

**Bruno Kahn (Paris):** *Reciprocity sheaves and motives with modulus*

I will give an update of this ongoing joint work with Shuji Saito and Takao Yamazaki.

**Nikita Karpenko (Edmonton):** *Chern subring for generic flag varieties*

For a split semisimple group  $G$ , a standard generic  $G$ -torsor  $E$  (over a field), and a parabolic subgroup  $P$  in  $G$ , we consider the question if the Chow ring of the generic flag variety  $E/P$  is generated by Chern classes.

**Bruno Klingler (Paris):** *Hodge theory and atypical intersections*

Given a smooth family of quasiprojective complex varieties  $f: X \rightarrow S$ , one would like to understand the locus of points  $s \in S$  whose fiber  $X_s$  admits more Hodge classes than the very general fiber. In this talk I will describe a general conjecture (and some results) concerning this locus in terms of exceptional intersections. When  $S$  is a Shimura variety and  $f: X \rightarrow S$  a standard family of Abelian varieties, one recovers the Zilber–Pink conjectures, in particular the André–Oort conjecture.

**Marc Levine (Essen):** *Enumerative geometry with quadratic forms*

We describe a program for using the Milnor–Witt sheaves and the oriented Chow groups of Barge–Morel to refine classical aspects of enumerative geometry to give identities in the Grothendieck–Witt group. This includes techniques for computing the refined Euler characteristic of a smooth projective variety, and refined versions of the classical formulas for counting degenerate fibers in a morphism of a smooth projective variety to a smooth projective curve. We also describe links of this theory to Morse theory.

**Niko Naumann (Regensburg):** *The  $\log_p$ -conjecture and higher blue-shift*

We determine the Balmer spectrum of the tensor-triangulated category  $SH(A)^\omega$  of finite genuine  $A$ -spectra, for a finite abelian group  $A$ . The case  $A = C_p$  is due to Balmer/Sanders. Using this result, we generalize a classical blue-shift result of Kuhn to the case of elementary abelian  $p$ -groups.

(On-going joint work with: Barthel, Hausmann, Nikolaus, Noel and Stapleton.)

**Alexander Schmidt (Heidelberg):** *Anabelian geometry with étale homotopy types*

(Joint work with J. Stix). Anabelian geometry with étale homotopy types generalizes in a natural way classical anabelian geometry with étale fundamental groups. We show that, both in the classical and the generalized sense, any point of a smooth variety over a field  $k$  which is finitely generated over  $\mathbb{Q}$  has a fundamental system of (affine) anabelian Zariski-neighbourhoods. This was predicted by Grothendieck in his letter to Faltings.

**Alexander Vishik (Nottingham):** *On subtle Stiefel–Whitney classes*

The subtle Stiefel–Whitney classes of quadratic forms (introduced by A. Smirnov and me) provide an important classification tool. These classes permit to describe the motive of the torsor (corresponding to a form), are related to the  $J$ -invariant of quadratic forms, and are expected to provide the description of the motivic cohomology ring of the Čech simplicial scheme of a torsor. I will discuss some computations with these classes.

**Claire Voisin (Paris):** *Universally defined cycles*

Universally defined cycles are those defined on families of smooth varieties of given dimension, and satisfying two natural functorial properties relative to base change and open inclusions. We prove that for dimension at most 3, universally defined cycles are the polynomials in the Chern classes of the relative tangent bundle. We also study the cycles universally defined on powers of smooth surfaces and show that they are polynomials in the diagonals and Chern classes of the relative tangent bundle. The motivation comes from the study of the punctual Hilbert scheme of a surface.

**Jörg Wildeshaus (Paris):** *Weights for representations and weights for motives*

Let  $G$  be a reductive group underlying Shimura data. Classically, one associated to any (algebraic, finite dimensional) representation  $V$  of  $G$  a variation of Hodge structure (in the Hodge theoretic context) or an  $l$ -adic sheaf (in the  $l$ -adic context) on the Shimura variety  $S$  associated to the Shimura data. This classical construction is referred to as the “canonical construction of sheaves” on  $S$  from representations of  $G$ .

Recently, ANCONA proved that the canonical construction of sheaves is motivic if the Shimura data are of PEL type (i.e., coming from moduli problems of Abelian varieties). That is, for any representation  $V$  of  $G$ , there is a relative Chow motive  $\mu(V)$  over  $S$ , whose (Hodge theoretic or  $l$ -adic) realizations give the classical canonical construction.

Let  $j$  denote the open immersion of  $S$  into its Baily–Borel compactification, and  $i$  the closed immersion of the complement of  $S$ . It is important to control the “degeneration to the boundary”  $i^*j_*$  of motives in the image of  $\mu$ . Examples suggest that the (motivic) weight of  $i^*j_*\mu(V)$  can be controlled if the (representation theoretic) weight of  $V$  is regular.

**Nobuaki Yagita (Ibaraki):** *Multiplicative structures of mod  $p$  Chow rings of versal complete flag varieties*

Let  $G$  be a compact Lie group, and  $G_k$  the split algebraic group over a field  $k$ . Moreover let  $E$  be a non-trivial  $G_k$ -torsor. We study a ring structure of mod  $p$  Chow ring  $\mathrm{CH}^*(X)/p$  for the complete flag variety  $X = E/B_k$ . For example, when  $(G, p) = (G_2, 2), (F_4, 3)$  or  $(E_8, 5)$ , we have a ring isomorphism

$$\mathrm{CH}^*(X)/p \cong \mathrm{CH}^*(BB_k)/(p, I(p)^2)$$

where  $I(p) = \mathrm{Ker}(\mathrm{CH}^*(BB_k)/p \rightarrow \mathrm{CH}^*(G_k/B_k)/p)$ . (Its additive structure is still known by Petrov–Semenov–Zainoulline.) For some other groups, there are similar (but different) isomorphisms.

**Kirill Zainoulline (Ottawa):** *From Hilbert bases for Laurent polynomials to cohomological invariants of linear algebraic groups*

The talk is based on the joint recent work by Baek, Devyatov and myself. We find explicit generators of the kernel of the characteristic map  $K_T(pt) \rightarrow K_T(G)$  where  $G = G^{sc}/\mu_2$  is the quotient modulo a central subgroup of the simply-connected semisimple split linear algebraic group  $G^{sc}$  and  $K_T(-)$  is the  $T$ -equivariant  $K_0$  with integer coefficients. Using these generators we compute (a) the  $K_0$  of a generic  $G$ -flag in terms of generators and a finite number of relations; (b) various subgroups of cohomological invariants of degree 3 of semisimple algebraic groups, hence, extending previous results by Merkurjev and others.

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**Participants**

Joseph Ayoub (Zürich)	Morten Lüders (Regensburg)
Tom Bachmann (Essen)	Hiroyasu Miyazaki (Tokyo)
Federico Binda (Regensburg)	Fabien Morel (LMU)
Luca Barbieri Viale (Milano)	Niko Naumann (Regensburg)
Denis-Charles Cisinski (Regensburg)	Alberto Navarro (Zürich)
Hélène Esnault (FU Berlin)	Manh Toan Nguyen (Osnabrück)
Mathieu Florence (Paris)	Stefano Nicotra (Liverpool)
Alberto Navarro Garmendia (Zürich)	Rakesh Pawar (Mumbai)
Stefan Gille (Edmonton)	Anand Sawant (LMU)
Pawel Gladki (Katowice)	Alexander Schmidt (Heidelberg)
Daniel Harrer (Essen)	Johannes Schmidt (Heidelberg)
Olivier Houton (LMU)	Stephen Scully (Edmonton)
Alexander Henke (LMU)	Pavel Sechin (Moscow)
Detlev Hoffmann (Dortmund)	Nikita Semenov (LMU)
Katharina Hübner (Heidelberg)	Florian Strunk (Regensburg)
Fangzhou Jin (Regensburg)	Alexander Vishik (Nottingham)
Bruno Kahn (Paris)	Claire Voisin (Paris)
Nikita Karpenko (Edmonton)	Jörg Wildeshaus (Paris)
Adeel Khan (Regensburg)	Heng Xie (Warwick)
Viktor Kleen (USC)	Nobuaki Yagita (Ibaraki)
Bruno Klingler (Paris)	Maria Yakerson (Essen)
Jonas Irgens Kylling (Oslo)	Feng Yue (Beijing)
Andrei Lavrenov (LMU)	Kirill Zainouline (Ottawa)
Florence Lecomte (Strasbourg)	Masoud Zargar (Princeton)
Marc Levine (Essen)	Maksim Zhykhovich (LMU)