Mathematisches Institut
der Universität München
Prof. Otto Forster

SoSe 2024
June 5, 2024

Selected Topics from Number Theory

Problem sheet #5

Problem 17

Let $I :=]0,1[\subset \mathbb{R}$ be the open unit interval. The coefficients of the CF expansion of $x \in I$,

$$x = \mathrm{cfrac}(0, a_1, a_2, \ldots)$$

can be viewed as functions $a_k(x)$ of the variable $x \in I$.
For $k = 1$ and $k = 2$ determine the points $t \in I$, where the function $a_k(x)$ is not continuous. (In this context a rational number $\xi \in I$ is considered as a point of discontinuity of $a_k(x)$, if the CF expansion of ξ ends before the index k).

Problem 18 (cf. Problem 9)

Let f_n be the Fibonacci numbers.

a) Prove that

$$\sum_{n=1}^{\infty} \frac{1}{f_n f_{n+2}} = 1.$$

b) Calculate the sum

$$A := \sum_{k=0}^{\infty} \frac{1}{f_{2^k}}$$

Problem 19

a) Prove: If the *Mersenne number* $M_k := 2^k - 1$ is prime, then k is a prime number.

b) Find the smallest prime number p such that M_p is not prime.

Problem 20 (Continuation of Problem 19)

Let $p \geqslant 3$ be a prime number and q be a prime divisor of M_p,

$$q \mid 2^p - 1.$$

Prove:

a) There exists a positive integer k such that $q = 2kp + 1$,

b) $q \equiv \pm 1 \bmod 8$.

These problems will be discussed Wednesday, June 19, 2024