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Foliations and Subriemannian Geometry, WS 02/03, (Prof. Kotschick)

Problem sheet 8

1. Let ∇ be an affine connection. Express ∇R through R and T. If the connection is torsion free, prove the second Bianchi identity

$$(\nabla_Z R)(X,Y)W + (\nabla_X R)(Y,Z)W + (\nabla_Y R)(Z,X)W = 0 \ .$$

2.

3. Let ∇ be a torsion free connection on M and $TM = V \oplus H$.

   a) Show that V is integrable if and only if $\nabla\mathcal{H}$ is symmetric on V, i.e. $(\nabla_X \mathcal{H})\, Y = (\nabla_Y \mathcal{H})\, X$ for all $X, Y \in V$.
   (Hint: One direction was treated in the lecture.)

   b) Prove directly that the condition "$\nabla\mathcal{H}$ is symmetric on V" is independent of the choice of the complement H if V is fixed.

4. Let $Y \in V$ and ∇ a torsion free connection. Give a complete proof of the identity

$$R(X,Z)Y = R^V(X,Z)Y + \left(\nabla^2_{X,Z}\mathcal{H} - \nabla^2_{Z,X}\mathcal{H}\right) Y + [\nabla_X\mathcal{H}, \nabla_Z\mathcal{H}]\, Y \ .$$

Please hand in your solutions during the lecture on Wednesday, 11.12.2002.