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Foliations and Subriemannian Geometry, WS 02/03, (Prof. Kotschick)

Problem sheet 7

1. a) Let D be a distribution on M and $X \in \mathfrak{X}(M)$. Show that the flow of X preserves D if and only if $[X, \mathcal{D}] \subset \mathcal{D}$.

   b) Let D be a fat distribution on M and $X \in D_p \setminus \{0\}$. Show that this vector can not be extended to a local vector field whose flow preserves D.

2. a) Let $\alpha \in \Omega^1(M)$ be a defining form for a contact structure. Show that there is a unique vector field X on M such that $\alpha(X) \equiv 1$ and $i_X d\alpha = 0$. This is the Reeb vector field of α.

   b) Let C be a contact structure on M and $X_p \in T_pM$ transversal to C_p. Show that X_p can be extended to a local vector field preserving C. (Hint : Fix a local defining form α such that $\alpha(X_p) = 1$ and look for a function g such that the Reeb vector field of $g\alpha$ at p is X_p.)

3. Consider a foliation $\mathcal{F}$ on M and $X \in \mathfrak{X}(M)$. Show that the following conditions are equivalent.

   a) The flow of X preserves $\mathcal{F}$, i.e. the flow maps leaves of $\mathcal{F}$ to leaves.

   b) X is basic.

   c) If $\varphi : U \to \mathbb{R}^q$ is a locally defined submersion whose fibers are the leaves of $\mathcal{F}\big|_U$, then $(D_p\varphi)(X)$ depends only on the leaf of $\mathcal{F}\big|_U$ through p.

4. Let ∇ be a connection on TM and $TM = H \oplus V$. For all local sections X, Y of TM we define

$$\begin{aligned} T_X Y &= \mathcal{H}\nabla_{\mathcal{V}X}(\mathcal{V}Y) + \mathcal{V}\nabla_{\mathcal{V}X}(\mathcal{H}Y) \\ A_X Y &= \mathcal{H}\nabla_{\mathcal{H}X}(\mathcal{V}Y) + \mathcal{V}\nabla_{\mathcal{H}X}(\mathcal{H}Y)\ . \end{aligned}$$

   Show that A and T are tensors of type $(2, 1)$.

   Now assume that ∇ is torsion free. Find compact formulae for ∇T and ∇A.

Please hand in your solutions during the lecture on Wednesday, 5.12.2002.