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Foliations and Subriemannian Geometry, WS 02/03, (Prof. Kotschick)

Problem sheet 3

1. Let $\alpha = dx + f(y)dz$. Consider the distribution $D = \ker(\alpha)$ on $\mathbb{R}^3$.
   a) For all smooth functions f describe the accessible set of $p \in \mathbb{R}^3$.
   b) Now suppose $f \equiv 0$ on $\{y \leq 0\}$ and $f(1) \neq 0$. Modify the distribution $\ker(\alpha)$ such that the accessible set is no longer $\mathbb{R}^3$. (Hint : Modify the distribution along a transversal line $\{(x, y_0, 0) | x \in \mathbb{R}\}$ with $y_0 < 0$.) What are the accesible sets now?

2. Calculate the nilpotentization of a distribution D in the following cases.
   a) Let D be an even contact structure, i.e. a distribution of codimension one on some manifold of dimension $2k$ whose intrinsic curvature has maximal rank.
   b) On $\mathbb{R}^4$ consider the one–forms $\alpha_1 = dz - xdy$ and $\alpha_2 = dx - wdy$. Let $D = \ker(\alpha_1) \cap \ker(\alpha_2)$.

3. Let ω_t be a smooth family of k–forms on some manifold M and let X_t be a time–dependent vector field. We denote the t–derivative of ω_t by $\dot{\omega}_t$. If ϕ_t is a local flow for X_t, show that

$$\left.\frac{d}{dt}\right|_{t=\tau} (\phi_t^* \omega_t) = \phi_\tau^* \left(L_{X_\tau} \omega_\tau + \dot{\omega}_\tau \right) \ .$$

4. Let M be a compact connected manifold of dimension n together with a volume form Ω.
   a) Prove that for every $p \in M$ there is a chart (U, φ) around this point such that in these coordinates $\Omega = dx_1 \wedge \cdots \wedge dx_n$.
   b) Let $\tilde{\Omega}$ be a volume form on M such that

$$\int_M \Omega = \int_M \tilde{\Omega} \ .$$

   Show that there is a diffeomorphism ϕ of M such that $\phi^* \Omega = \tilde{\Omega}$. (Hint : Adapt the method used in lecture for the proof of the Darboux theorem for symplectic forms.)

The **exercise classes** will take place on Thursdays, 9–11 in room E46.

Please hand in your solutions during the lecture on Wednesday, 6.11.2002.