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Foliations and Subriemannian Geometry, WS 02/03, (Prof. Kotschick)

Problem sheet 14

1. Let V be an oriented foliation of dimension one on a compact manifold. Let $\omega_t \in \Omega^1(M)$ be a smooth family of closed forms such that

   (i) $\omega_t\big|_V > 0$

   (ii) $[\omega_t] \in H^1(M)$ is constant.

   Use the foliated Moser method in order to construct a family of diffeomorphisms φ_t which preserve V and $\varphi_t^*\omega_t = \omega_0$.

2. Let μ be the Riemannian volume form of the standard metric on $S^2 \subset \mathbb{R}^3$. Consider

$$\begin{aligned} \psi_t : S^2 &\longrightarrow S^2 \\ (x, y, z) &\longmapsto (\cos(t)x + \sin(t)y, -\sin(t)x + \cos(t)y, z) \ . \end{aligned}$$

   Let V be the foliation on $S^1 \times S^2$ induced by the second factor. Consider $[0, 2\pi] \times S^2/\sim$ with $(1, p) \sim (0, \psi_t(p))$ and the foliation $\mathcal{F}_t$ induced by the first factor. In particular, $\mathcal{F}_t$ is a complement of V in $T(S^1 \times S^2)$.

   (a) Use μ in order to construct a smooth family ω_t of closed defining forms for $\mathcal{F}_t$ such that $[\omega_t] \in H^2(S^1 \times S^2)$ is independent of t. (Note that $\mathcal{F}_t$ is volume–preserving.)

   (b) Show that the foliations $\mathcal{F}_t$ are not all diffeomorphic.

3. Let α be a contact form and Y the Reeb vector field of α. If β is a 1–form with $\beta(Y) = 0$, then prove that there is a unique vector field X such that $\alpha(X) = 0$ and $\beta = i_X d\alpha$.

4. This is the last exercise for this course. Those of you who want a "Schein" should provide the following information : name, date of birth, place of birth, address.

Please hand in your solutions during the lecture on Wednesday, 6.2.2003.