Mathematisches Institut
Universität München

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Office hour: Mon 1-2 pm, Room 308 SS 04

## Geometry of Manifolds II (Prof. Kotschick)
## Problem sheet 8

1. Let $(M,g)$ be a connected Riemannian manifold and $p \in M$ a point. Show that a Killing field $X$ is uniquely determined by $X(p)$ and $(\nabla X)(p)$.

2. Let $L(p,q), L(p',q')$ be lens spaces as defined in Ex. 4 on sheet 5 with the induced metric of constant curvature $K=1$. What can one say about the fundamental group and positivity of curvature on the product $L(p,q) \times L(p',q')$? Why does this not give a counterexample to the Theorem of Synge?

3. Let $(M,g)$ be an oriented Riemannian manifold of positive sectional curvature and even dimension. Let $\gamma$ be a closed geodesic in $M$, i.e. an immersion of $S^1$ in $M$ which is geodesic at all of its points. Prove that every $\epsilon$-neighbourhood of $\gamma$ contains a closed curve $c$ homotopic to $\gamma$ such that the length of $c$ is strictly less than that of $\gamma$.

4. Prove the following theorem of Zassenhaus: if $G$ is a finite abelian group which acts isometrically and freely on $S^n$ with the standard metric, then $G$ is cyclic.

Hint: Show that a non-cyclic finite abelian group contains a subgroup $H$ isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$ for a prime $p$. Consider $\sum_{h \in H} h \cdot v \in \mathbb{R}^{n+1}$ for $v \in S^n$ and the set of all proper subgroups of $H$.

Please hand in your solutions on Monday, the 28th of June, in the lecture.