Mathematisches Institut
Universität München

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Office hour: Mon 1-2 pm, Room 308 WS 03/04

Geometry of Manifolds I (Prof. Kotschick)
Problem sheet 12

1. For the following vector fields on $\mathbb{R}^2$ sketch the flow lines and determine the index of the singularity at $(x, y) = (0, 0)$.

$$a)\, V(x,y) = (-y, x) \quad b)\, V(x,y) = (x, -y) \quad c)\, V(x,y) = (x^2 - y^2, 2xy).$$

2. For every integer $n \in \mathbb{Z}$ construct a vector field on $\mathbb{C} \cong \mathbb{R}^2$ which has a singularity of index n at $z = 0$. Sketch the flow lines for $n = \pm 2$ and $n = \pm 3$.

3. Let $M = \{(x, y, x^2 - y^2) \mid x, y \in \mathbb{R}\} \subset \mathbb{R}^3$. This is an open 2-dimensional submanifold of $\mathbb{R}^3$. Show that the Gauss curvature for the induced Riemannian metric is negative at every $p \in M$.

4. Let $M \subset \mathbb{R}^3$ be a closed 2-dimensional submanifold with the induced Riemannian metric. Show that there is a point $p \in M$ where the Gauss curvature is strictly positive.
(Hint: Define the smooth function $f : M \to \mathbb{R}, x \mapsto ||x||^2$, where $||x||$ is the Euclidean distance of a point $x \in M$ from the origin, measured in $\mathbb{R}^3$. Consider a point where this function assumes its maximum.)

Please hand in your solutions on Thursday, the 29th of January, in the lecture.