Mathematisches Institut
Universität München

Jonathan Bowden
Office hour: Wed 2-3 pm, Room 308 WS 06/07

Geometry of Manifolds I (Prof. Kotschick)
Problem sheet 4

1. Find the flow on $\mathbb{R}^2$ determined by the following vector fields.
a) $V = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$
b) $V = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$

2. a) Consider $B_2(0) \subset \mathbb{R}^n$. For any $y \in B_1(0)$ construct a vector field on $B_2(0)$ which is complete, ie has a well defined global flow Φ_t, and $\Phi_1(0) = y$.

b) Use this to show that for a connected, differentiable manifold M and any two given points $x, y \in M$, there is a diffeomorphism $\phi : M \to M$ such that $\phi(x) = y$.

3. Let M be a compact, differentiable manifold, using local charts and a partition of unity, show that there exists a smooth map

$$f : M \to \mathbb{R}^N$$

for some sufficiently large N, which is injective and a topological embedding.

4. Construct two distinct flows on the 2-torus $M = S^1 \times S^1$ with the following properties:
a) All of its flow lines are closed.
b) None of the flow lines are closed.
You may use the desription of the torus as $M \cong \mathbb{R}^2/\mathbb{Z}^2$

Please hand in your solutions on Friday the 17th of November by 4 PM, in the appropriate box.