Mathematisches Institut
Universität München

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Office hour: Wed 2-3 pm, Room 308 WS 06/07

Geometry of Manifolds I (Prof. Kotschick)
Problem sheet 1

1. Recall that a topological space X is called *compact* if every open covering of X has a finite subcovering. A subset of a topological space is called compact if it is compact in the subspace topology.
Let X and Y be topological spaces. Prove:
 a) If X is compact and $A \subset X$ closed, then A is compact.
 b) If $f : X \to Y$ is continuous and X compact, then $f(X) \subset Y$ is compact.
 c) If X is Hausdorff and $A \subset X$ compact, then A is closed.

2. Let M be a topological manifold. Show that there exists a countable basis of the topology consisting of open sets with compact closures (note that the definition of a manifold asks for a countable basis without other conditions).

3. Let M and N be differentiable manifolds. Define a differentiable structure on the cartesian product $M \times N$. Show that the projection $\pi : M \times N \to M, (x, y) \mapsto x$ and the injection $i : M \to M \times N, x \mapsto (x, p)$, for some fixed $p \in N$, are differentiable mappings.

4. The n-sphere S^n is defined as the subspace $\{x \in \mathbb{R}^{n+1} | x_1^2 + ... + x_{n+1}^2 = 1\}$ of $\mathbb{R}^{n+1}$. Consider the north-pole $N = (0, 0, ..., 0, 1) \in S^n$. The *stereographic projection* from N is the map $\phi_N : S^n \backslash N \to \mathbb{R}^n$ which takes a point $p \in S^n \backslash N$ to the intersection of the line through N and p with the hyperplane $\mathbb{R}^n \times 0 \subset \mathbb{R}^{n+1}$. There is a similar map ϕ_S for the south-pole $S = (0, 0, ..., 0, -1)$.

Write down coordinate expressions for ϕ_N and ϕ_S and show that they make S^n into a compact n-dimensional differentiable manifold.

Please hand in your solutions by Wednesday, the 25th of October 4 pm, in the Übungskasten.