Length minimizing geodesics of the Hofer's metric

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We will discuss sufficient conditions for a Hamiltonian function to generate a length minimizing geodesic in the Hamiltonian diffeomorphism group of a symplectic manifold endowed with the Hofer's metric.
We will consider the case of the standard symplectic space ($\mathbb{R}^{2n}$,ω_0) and the case of a closed symplectic manifold such that the class of the symplectic form and the first Chern class both vanish on the second homotopy group.
In both cases results can be obtained by comparing the Hofer's metric with Viterbo's in the $\mathbb{R}^{2n}$ case and its generalisation by Schwarz in the case of a closed manifold.
Our main result states that a geodesic arc generated by a non autonomous Hamiltonian function $H : [0,1] \times M \to \mathbb{R}$, is minimizing provided that the Hamiltonian vector field associated to H has no degenerate periodic orbit of period smaller than one. If that is the case the distances between the endpoints of the geodesic arc in Hofer's and Viterbo's metric coincide.