LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

MATHEMATISCHES INSTITUT

Fall term 2022

Prof. D. Kotschick
Dr. J. Stelzig

Selected Topics in Complex Geometry

Sheet 01

Exercise 1.

1. Show that the Fubini-Study form ω_{FS} (defined in the lecture) is indeed well-defined and a Kähler form.
2. Show that the form $\omega = \frac{1}{2i}\partial\bar{\partial}(1 - \|z\|^2)$ is a Kähler form on $D^n = \{z \in \mathbb{C}^n \mid \|z\| < 1\}$.

Exercise 2. Prove that on any compact complex manifold X, the Kähler cone $K_X \subseteq H^{1,1}(X, \mathbb{R})$ is an open convex cone.

Exercise 3. Show that a hermitian metric h on a complex manifold X, with associated two-form $\omega = -\operatorname{Im}(h)$, is Kähler if and only if around every point $x \in X$ there is a local coordinate system $z_1, ..., z_n$ for which the Taylor-expansion of $\omega - \sum_{i_1}^{n} dz_i \wedge d\bar{z}_i$ has no linear term in the $z_i, \bar{z}_i$.

Exercise 4. Show that a hermitian metric h on a complex manifold X, with associated two-form ω and associated Riemannian metric $g = \operatorname{Re}(h)$, is Kähler if and only if J is parallel for the associated Levi-Cività connection, i.e. $\nabla J = 0$.

Submit your solutions to J. Stelzig by email or on paper until Wed, May 4th, 14:00.