Korn type inequalities in Orlicz spaces

Many problems in the mathematical theory of Generalized Newtonian fluids and in the mechanics of solids lead to the following question: it is possible to bound a suitable energy depending on ∇u by the corresponding one in dependence on $\varepsilon(u) := \frac{1}{2}\left(\nabla u + \nabla u^T\right)$, i.e.,

$$\int_\Omega |\nabla u|^p \, dx \le c(p,\Omega) \int_\Omega |\varepsilon(u)|^p \, dx$$

for all $u \in \mathring{W}^{1,p}(\Omega, \mathbb{R}^d)$? This is true for all $1 < p < \infty$. We discuss generalizations of Korn's inequality above to Orlicz spaces and their applications. Moreover, we consider Korn-type inequalities involving the trace-free part of the symmetric gradient

$$\varepsilon^D(u) := \varepsilon(u) - \frac{1}{d}\mathrm{div} u \, I$$

and corresponding variational integrals.