Harmonic N-particle systems: reduced density operators and their properties

Solving analytically the N-particle Schrödinger equation for interacting particles is typically impossible. One exception is given by harmonic models which are also relevant from a physical viewpoint since they arise as an effective description of lattice systems. We present two results. First, we prove that for any eigenstate of a harmonic system each of its M-particle reduced density operators ρ_M obeys a duality condition. This condition implies duality relations for the eigenvalues λ_k of ρ_M and relates a harmonic model with length scales $l_1, \ldots, l_N$ with another one with inverse lengths $1/l_1, \ldots, 1/l_N$. Entanglement entropies and correlation functions inherit duality from ρ_M. Second, for the specific case of N identical particles we explore the influence of the exchange statistics on the 1-particle properties obtained from ρ_1. Although the (natural) occupation numbers for fermions and bosons differ significantly the fermionic and bosonic natural orbitals are very similar.