



# What Bell Did

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# The First Fundamental Fact About Bell's Theorem

- Most physicists do not know what Bell proved.
- More than, that, most physicists have the opposite of understanding of what Bell proved.

# Second Fundamental Fact About Bell's Theorem

- Most physicists do not understand what Bell proved because
- 1) they do not understand what Einstein, Podolsky and Rosen proved in the EPR paper, and
- 2) Bell begins his paper assuming that the reader has understood the EPR paper.

# Third Fundamental Fact, Which Lies Behind 1) and 2)

- Most physicists do not understand even what bothered *Einstein* about quantum theory.
- All of these assertions require some evidence.



# Evidence

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## Why did Einstein say 'God doesn't play dice'?

Mar 4, 2013 5 comments

[Link to the web page of the video](#)

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# What Bell Said About “Hidden Variables”

- “Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism are not forced on us by experimental facts, but by deliberate theoretical choice?” (SUQM p. 160)

# What Einstein Said About Quantum Theory

- “It seems hard to sneak a look at God’s cards, but that he plays dice and uses “telepathic” methods (as the present quantum theory requires of him) is something I cannot believe for a moment.”

# What Pauli Said to Born

- "...I was unable to recognize Einstein whenever you talked about him in either your letter or your manuscript. It seemed to me as if you had erected some dummy Einstein for yourself, which you then knocked down with great pomp. In particular, Einstein does not consider the concept of 'determinism' to be as fundamental as it is frequently held to be (as he told me emphatically many times)...he disputes that he uses as a criterion of admissibility of a theory "Is it rigorously deterministic?"...he was not at all annoyed with you, but only said you were a person who will not listen."

# EPR Neat

- The heart of the EPR argument relies on a *locality principle*, even though EPR do not put it explicitly that way.
- The locality principle comes into play when EPR apply their *criterion for an element of physical reality*.
- The criterion itself is analytic, and so is not the sort of thing one can coherently dispute. But one can dispute, in a certain case, whether the criterion applies.

# What EPR Said

- “The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one such way, whenever the conditions set down in it occur. Regarded not as a necessary, but merely as a sufficient condition of reality, this criterion is in agreement with classical as well as quantum-mechanical ideas of reality.”

# Why It is Analytic

- To “disturb” a system means to alter its physical state.
- So if a procedure does *not* disturb a system, then whatever physical characteristics can be determined by use of the procedure *must already be physical characteristics of the system*.
- Further, if one even can determine a physical characteristic of a system without disturbing it, then the system must already have that characteristic, i.e. it must be an element of physical reality.

# Therefore

- The issue with the EPR argument is not whether the criterion they announce is *correct*, but rather just whether, in the experiment they describe it *applies*.
- In order to apply, a procedure used to determine a physical characteristic of a system must be regarded as *not disturbing the system*.
- EPR try to guarantee this lack of disturbance by using *remoteness in space-time* as an *insulator against disturbance*.
- This is the EPR doctrine of locality.

# EPR-Locality

- A physical theory is *EPR-local* iff according to the theory procedures carried out in one region do not immediately disturb the physical state of systems in sufficiently distant regions in any significant way.

# Denial of EPR-Locality

- A physical theory is not *EPR-local* iff according to the theory some procedures carried out in one region do immediately disturb the physical state of systems in extremely distant regions in a significant way.
- It would be reasonable to call such a disturbance “spooky action-at-a-distance” (“*spukhafte Fernwirkung*”), which is the complaint Einstein raised about standard *quantum theory*.

# Historical Comment

- Every “classical” physical theory before quantum theory, even *Newtonian gravitational theory* was EPR-local as just defined.
- This came about for two reasons.

# Reason 1: Spatial Attenuation

- Consider Newtonian gravity, postulated as acting instantaneously and at-a-distance.
- (Newton would not have so regarded it.)
- Since the gravitational force falls off as the inverse square of the distance, separating two systems by a great enough distance means that no action on the one will have a *significant* gravitational effect on the other: no procedure in one place gravitationally disturbs sufficiently distant systems.

## Reason 2: Time Lag

- A second way that spatial distance creates isolation in classical physics is via time lag: changes in one location typically have to propagate to distant locations, which takes time.
- (Newton would have expected this for gravity.)
- If so, then no procedure carried out on one system can *immediately* change the state of a sufficiently distant system.

# EPR in one step

- If a theory is EPR local, and the outcomes of any pair of experiments done arbitrarily far apart are guaranteed by the theory to be perfectly correlated, then there must be an element of reality pertaining to each system that determines the outcome.
- For, by experiments on one system one can determine how the other will behave.
- But by EPR locality, the experiment did not disturb or alter the state of the distant system.
- Therefore something in its physical state determines the outcome.

# Forget Complementarity

- In the EPR setting, this argument already shows just by considering position measurements alone that *if the actual physics of the world is EPR-local, then the quantum description of a system is incomplete.*
- For quantum mechanics predicts the right sort of correlations between the outcome of position measurements.
- If the the world is EPR-local, the measurement on one side does not disturb the state on the other.
- Therefore, there is a position-measurement element-of-reality in the distant system *not reflected in its quantum state.*

# Also Momentum

- In the original EPR state, the perfect anti-correlation of momentum provides an analogous argument that there must be momentum-measurement elements-of-reality that determine the outcomes of “momentum measurements”, assuming the actual physics is EPR local.
- Hence, an EPR-local physics must violate complementarity. But that is an aside.

# EPR Dilemma

- The EPR argument leaves us with a dilemma: if the predictions of the quantum formalism are accurate, then either the quantum description of the world is incomplete or the actual physics is not EPR-local.
- Bohr and company insisted on the completeness of the quantum description, and so committed themselves to denying EPR-locality.
- Hence Einstein's remark that in the standard account God both plays dice (since there is no pre-existing element of reality that determines the outcome) and uses telepathic methods.

# Schrödinger

- Schrödinger immediately understood the import of EPR, and extended the observation to maximally entangled pairs where the value of every observable associated with one system can be determined by an appropriate procedure carried out on the other.
- If the world is EPR-local, then the outcome of any “measurement” carried out on one must be predetermined by an EPR-local element of reality that is unaffected by experiments carried out on the other.

# Schrödinger

- “But let us once more make the matter very clear. Let us focus attention on the system labeled with the small letters p, q and call it for brevity the “small” one. Then things stand as follows. I can direct one of two questions at the small system, either that about q or that about p. Before doing so I can, if I choose, procure the answer to one of these questions by a measurement on the fully separated other system (which we may regard as auxiliary apparatus), or I may intend to take care of this afterward. My small system, like a schoolboy under examination, cannot possibly know whether I have done this or for which questions, or whether and for which I intend to do it later. From arbitrarily many pretrials I know that the pupil will correctly answer the first question that I put to him. From that it follows that in every case he knows the answer to both questions. That the answering of the first question so tires or confuses the pupil that his further answers are worthless changes nothing at all of this conclusion.”

# Schrödinger's Dilemma

- Schrödinger notes that by making a position measurement on one side and a momentum measurement on the other, and using the strict correlation/anti-correlation, one can ascribe both position and momentum to each system.
- “There is no doubt about it. Every measurement is for its system the first. Measurement on separated systems cannot directly influence each other—that would be magic. Neither can it be by chance, if from a thousand experiments it is established that virginal measurements agree.
- The prediction catalog  $q = 4, p = -7$  would of course be hypermaximal.”

# Bohm's Spin Version

- Although Schrödinger proved *in principle* that the outcome of a “measurement” of any “observable” on one side of an entangled pair could be foretold by an appropriate experiment on the other, most of these “observables” for the EPR state are unfamiliar, and the two that are not (position and momentum) turn out to be uncorrelated.
- Bohm's recasting of the situation in terms of spin eliminated that feature, since the various spins are easily measured. This facilitates asking about correlations between non-aligned and non-orthogonal spin directions.

# Recap

- Einstein's main complaint about "standard quantum mechanics" (i.e. Copenhagen or now "operational" quantum theory) was not that it is *indeterministic* but that it requires a *form of non-locality* (spooky action-at-a-distance).
- This form of non-locality in the standard theory has nothing to do with *sending signals faster than light*. Einstein never imagined the EPR situation allowed one to do that: indeed it is obvious that one cannot use the correlations that EPR mention.

# Recap Con't

- The EPR correlations *per se* do not require such action-at-a-distance. The phenomena can be recovered without it, but only by a deterministic theory.
- This is the point of Bertlmann's socks.
- Einstein anticipated that one could find an EPR local theory that recovers *all* the predictions of quantum theory. It would have to be deterministic in EPR settings.
- Although Einstein seemed to prefer both EPR-locality and determinism, EPR-locality was more important to him.

# Finally...Bell

- Bell had some advantages over Einstein. Bohm's 1952 had proven that it is, after all, possible to reconcile the empirical predictions of quantum theory with determinism. But the non-locality of the pilot wave theory was manifest...
- "So in this theory an explicit causal mechanism exists whereby the disposition of one piece of apparatus affects the results obtained with a distant piece. In fact the Einstein-Podolsky-Rosen paradox is resolved in the way which Einstein would have liked least." SUQM . P. 11

# On the Einstein-Podolsky-Rosen Paradox

- “The paradox of Einstein, Podolsky and Rosen was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality. In this note, that idea will be formulated and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely, that the result of an experiment on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty.”

# Presuppositions of the Proof

- 1) Accuracy of Quantum Predictions
- 2) EPR Locality
- 3) Random Sampling
- From 1 & 2, via EPR, the theory must postulate “elements of reality” that predetermine the results of experiments on one system independently of what experiments are carried out on the other.
- From 3, the observed statistics of the outcomes must be (almost) the actual statistical distribution of these elements of reality.

# Main Mathematical Observation

- No possible distribution of EPR-local elements of reality can have the observed statistics.

# For Example

- In polarization experiments done on appropriately entangled photons, if the polarizers are misaligned by an angle  $\theta$ , then the photons will either both pass or both be absorbed by the polarizer  $\cos^2(\theta)$  of the time.
- So photons agree 100% of the time when the polarizers are aligned, 75% of the time when misaligned by  $30^\circ$ , and 25% of the time when misaligned by  $60^\circ$ .
- By the EPR argument, this behavior must be predetermined by an EPR-local element of reality (given presupposition 2).



**But...**

- ## ■ Impossible!

# GHZ

- A triple of particles are created in an entangled state and allowed to separate. Each will be subjected to either an x-spin “measurement” or a y-spin “measurement”.
- Prediction: If 2 X-spins and a Y-spin are measured, there will be an even number of “up” results.
- If all 3 Y-spins are measured, there will be an odd number of “up” results.
- If the theory is EPR-local, the result of each possible experiment must be predetermined for each particle irrespective of what is done to the others.

Impossible!



# Conclusion

- No EPR-local theory can replicate the predictions of the quantum formalism for experiments done far away from each other (at space-like separation), so....
- If the predictions of the quantum formalism for such experiments are accurate, *the physical world itself cannot be EPR-local*, so...
- In the precise sense that bothered Einstein, there is spooky action-at-a-distance.