Algebraic Number Theory

Exercises 2

Dr. Tom Bachmann Winter Semester 2021/2

Exercise 1. Let A be an integrally closed integral domain and $S \subset A$ be a multiplicatively closed subset. Show that $S^{-1}A$ is also integrally closed.

Exercise 2. Let A be an integral domain with field of fractions K, L/K a finite extension and $S \subset A$ be a multiplicatively closed subset. Show that

$$\overline{S^{-1}A}_L = S^{-1}(\overline{A}_L).$$

Exercise 3. Let $d \in \mathbb{Z}$ be squarefree and $K = \mathbb{Q}(\alpha)$, where $\alpha^2 = d$.

(1) Describe the kernel and image of $Tr_{\mathbb{Z}}^{\mathcal{O}_K} : \mathcal{O}_K \to \mathbb{Z}$.

(2) If $d < 0$ show that $N_{\mathbb{Z}}^{\mathcal{O}_K} : \mathcal{O}_K^\times \to \{\pm 1\}$ is *trivial*. Show that $N_{\mathbb{Z}}^{\mathcal{O}_K}$ is onto if $d = 2$, and trivial if $d = 3$.

Exercise 4. Let K as in Exercise 3. Compute the discriminant $d_K \in \mathbb{Z}$.