Algebraic Number Theory

Exercises 1

Dr. Tom Bachmann Winter Semester 2021/2

Exercise 1. Let A be an integral domain with fraction field K, L/K an algebraic field extension and B the integral closure of A in L. Show that L is the field of fractions of B.

Exercise 2. Let $A \subset B$ be an integral extension of integral domains. Show that A is a field if and only if B is a field.

Exercise 3. Let $A \subset B \subset C$ be integral extensions. Show that $A \subset C$ is also an integral extension.

Exercise 4. Let $K/\mathbb{Q}$ be a finite extension and $x \in K$. Show that x is integral if and only if the (monic) minimal polynomial of x over $\mathbb{Q}$ has coefficients in $\mathbb{Z}$.

Exercise 5. Show that $\mathcal{O}_K$ is a PID, for $K = \mathbb{Q}(i\sqrt{3})$ and $K = \mathbb{Q}(i\sqrt{2})$.