Algebraic Geometry 2

Exercises 9

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Exercise 1. Give an example of a morphism $f : X \to Y$ of finite type k-schemes and M a coherent sheaf on X such that $f_* M$ is not coherent.

Exercise 2. Let X be a noetherian topological space.
   (1) Show that if F is a presheaf on X satisfying the sheaf condition for *finite* coverings, then F is a sheaf.
   (2) Deduce that if $\{M_i\}_{i \in I}$ is a (possibly infinite) family of sheaves of abelian groups of X, then the presheaf
$$U \mapsto \bigoplus_{i \in I} M_i(U)$$
   is a sheaf.

Exercise 3.
   (1) Describe the category of $\mathcal{O}_X$-modules for $X = \operatorname{Spec}(A)$, A a dvr. Construct an $\mathcal{O}_X$-module which is not quasi-coherent.
   (2) Repeat question (1) where now A is a Dedekind domain.