Algebraic Geometry 2

Exercises 7

Dr. Tom Bachmann — Summer Semester 2021

Exercise 1. Let K/k be a finite type field extension of transcendence degree 1 and $k \subset R \subset K$ a valuation ring with field of fractions K. Show that there exists a Dedekind domain $k \subset A \subset R$ of finite type over k and $P \in \operatorname{Spec}(A)$ such that $R = A_P$. (In particular R is a dvr.)

Exercise 2. Let C be a proper regular curve over the field k. Show that the assignment

$$\{\varphi : C \to \mathbb{P}^1_k \mid \varphi \text{ finite surjective}\} \to \{f \in k(C) \mid f \text{ transcendental over } k\},$$
$$\varphi \mapsto \varphi|_{\varphi^{-1}(\mathbb{A}^1)}$$

is a well-defined bijection. In particular show that the left hand set is non-empty.

Exercise 3. Let X, Y be proper regular curves over the field k. Show that if $k(X) \simeq k(Y)$ as k-algebras, then $X \simeq Y$ as k-schemes.

Exercise 4. Let X be an affine curve over k and $x \in X$ closed. Show that $X \setminus \{x\}$ is affine.

[*Hint*: first reduce to the case where X is regular. (Both the regular case and the reduction are somewhat involved.)]