Algebraic Geometry 2

Exercises 6

Dr. Tom Bachmann Summer Semester 2021

Exercise 1. Let A be an integral domain of finite type over a field. Show that A is integrally closed if and only if A_P is a dvr whenever $P \in \operatorname{Spec}(A)$ has height 1, and

$$A = \bigcap_{ht(P)=1} A_P.$$

[*Hint:* for the last part, reduce to the case of UFDs using noether normalization and going up/down.]

Exercise 2. Let $f : C \to D$ be a birational morphism of curves over the field k. Show that if D is regular then f is an open immersion.

Exercise 3. Let $f : C \to D$ be a dominant morphism of proper curves over the field k. Show that if C is regular then f is finite and surjective.

[*Hint:* consider the normalization of D in $k(C)$.]

Exercise 4. Let C be a proper, regular curve over the field k and $U \subset C$ a non-empty open subscheme. Let X be a proper scheme over k and $f : U \to X$ a k-morphism. Show that f extends uniquely to C.

[*Hint:* consider the closure of the graph of f in $X \times C$.]