Algebraic Geometry 2

Exercises 5

Dr. Tom Bachmann Summer Semester 2021

Exercise 1. Let A be a unique factorization domain. Show that A is integrally closed (in its field of fractions).

Exercise 2. Let k be a field.

(1) Show that $k[T]$ is a Dedekind domain. (Equivalently, $\mathbb{A}^1_k$ is a regular curve over k.)
(2) Show that $k[X,Y]/(Y^3 - X^2)$ is *not* a Dedekind domain.
(3) Show that the homomorphism $k[X,Y]/(Y^3 - X^2) \to k[T]$ sending X to T^3 and Y to T^2 is an integral closure (in other words it is an injection of integral domains such that the target is the integral closure of the source in the fraction field of the target).