Algebraic Geometry 2

Exercises 3

Dr. Tom Bachmann Summer Semester 2021

Exercise 1. Let $S_\bullet$ be a commutative $\mathbb{N}$-graded ring and $f \in S_0$. Let $D_h(f) \subset \mathrm{Proj}(S_\bullet)$ denote the subset of homogeneous primes not containing f.

Show that $D_h(f)$ is an open subset, and $D_h(f) \simeq \mathrm{Proj}((S_\bullet)_f)$ as schemes.

Exercise 2. Let $S_\bullet$ be a commutative $\mathbb{N}$-graded ring and $d \geq 1$. Put

$$S_\bullet^{(d)} = \bigoplus_n S_{nd}.$$

Show that $S_\bullet^{(d)}$ is a commutative $\mathbb{N}$-graded ring and $\mathrm{Proj}(S_\bullet^{(d)}) \simeq \mathrm{Proj}(S_\bullet)$ as schemes.

Exercise 3. Let $\varphi : S_\bullet \to T_\bullet$ be a morphism of commutative $\mathbb{N}$-graded rings. Put

$$U = \{q \in \mathrm{Proj}(T_\bullet) \mid \varphi(S_+) \not\subset q\}.$$

Show that $U \subset \mathrm{Proj}(T_\bullet)$ is an open subset and construct an induced morphism of schemes

$$\mathrm{Proj}(\varphi) : U \to \mathrm{Proj}(S_\bullet).$$

Exercise 4. Notation as in the previous exercise. Suppose that φ is surjective. Show that $U = \mathrm{Proj}(T_\bullet)$ and $\mathrm{Proj}(\varphi)$ is a closed immersion.