1. $D_h(f) \cap D_+(g)$ = {primes not ctg. g or f}

$g \in S_0$ homog. $\overset{(1)}{\cong} D^{S_{(g)}}(f) \subset \operatorname{Spec} S_{(g)}$

$\overset{(2)}{\cong} D_+^{S_f}(g) \subset \operatorname{Proj} S_f$

Since $D_+(g) \subset \operatorname{Proj}(S)$ is open, $D_h(f)$ is open by (1).
& such of form $D_+(g)$ cover

Since $\operatorname{Proj} S_f$ is glued from the opens $D_+^{S_f}(g)$,
$\operatorname{Proj}(S_f) \cong D_h(f)$ by (2).

2. Clearly $S^{(d)}$ is comm. $\mathbb{N}$-graded ring.

Define maps $\operatorname{Proj} S \underset{\psi}{\overset{\varphi}{\rightleftarrows}} \operatorname{Proj} S^{(d)}$

$P \longmapsto P \cap S^{(d)}$

$\{x \in S \mid x^d \in Q\} \longmapsfrom Q$

Show ψQ is prime: $ab \in \psi Q \Rightarrow (ab)^d \in Q$

$\Rightarrow a^d \in Q$ or $b^d \in Q$

$\Rightarrow a \in \psi Q$ or $b \in \psi Q$.

Clear: $\psi\varphi P \subset P$

Conversely: $x \in P \Rightarrow x^d \in \psi\varphi P \Rightarrow x \in \psi\varphi P$.

$\therefore \psi\varphi P = P$

show $\varphi\psi Q = Q$: $x \in \varphi\psi Q \Leftrightarrow x^d \in Q \Leftrightarrow x \in Q$.

Hence: φ, ψ inverse bij^n.

Note: identify $D_+(f)$ & $D_+(f^d)$

$\therefore$ inverse homeom^m.

Note: $S_{(f)} \cong S^{(d)}_{(f^d)}$

$\therefore$ iso of sh.

3. $\varphi(S_+) \not\subset q \Leftrightarrow \varphi(s) \not\in q$ for some $s \in S_+$ homog.

$\therefore$ $U = \bigcup_{\substack{s \in S_+ \\ \text{hom.}}} D_+(\varphi(s))$ is open.

Have map $S_s \xrightarrow{\varphi} T_{\varphi(s)}$

$\rightsquigarrow$ $S_{(s)} \longrightarrow T_{(\varphi(s))}$

$\rightsquigarrow$ $D_+(\varphi(s)) \longrightarrow D_+(s)$.

Compatible with loc^n
$$\begin{pmatrix} S_s & \to & T_{\varphi(s)} \\ \downarrow & & \downarrow \\ S_{st} & \to & T_{\varphi(st)} \end{pmatrix}$$

$\therefore$ glue

$\therefore$ obtain $U = \bigcup D_+(\varphi(s)) \longrightarrow \operatorname{Proj} S$.

4. φ surj. $\Rightarrow$ $\varphi(S_+) = T_+$ $\Rightarrow U = \operatorname{Proj} T$.

By construction $\exists$ open cover of source & target s.t. restricted map is closed imm.

$\therefore$ φ is an immersion

$\therefore$ φ is closed imm. $\Leftrightarrow$ $\varphi(\mathrm{Proj}\,T) \subset \mathrm{Proj}(S)$ is closed.

Note that $\varphi : U \longrightarrow \mathrm{Proj}(S)$ is.

$$q \longmapsto \varphi^{-1}(q)$$

$\therefore$ Image of φ is $V_+(K)$, $K = \ker(S \to T)$.

— this is closed.