

# Algebraic Geometry 2

## Exercises 2

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**Exercise 1.** Let  $i : X \rightarrow Y$  be an open immersion. Show that  $i$  is proper if and only if  $Y \simeq X \amalg Z$ .

**Exercise 2.** Show that  $f : X \rightarrow Y$  is separated if and only if the image of the diagonal  $X \rightarrow X \times_Y X$  is a closed subset.

**Exercise 3.** Let  $f : X \rightarrow Y$  be a quasi-compact morphism of schemes (i.e. the preimage of any quasi-compact open subset is quasi-compact). Show that  $f(X) \subset Y$  is closed if and only if whenever  $y \in f(X)$  then also  $\overline{\{y\}} \subset f(X)$  (i.e.  $f(X)$  is stable under specialization).

**Exercise 4.** Let  $S$  be a scheme,  $X$  a reduced  $S$ -scheme and  $Y$  a separated  $S$ -scheme. Suppose given  $S$ -morphisms  $f, g : X \rightarrow Y$  agreeing on a dense open subset of  $X$ . Show that  $f = g$ .

Show by example that neither the assumption on  $X$  nor on  $Y$  can be removed.