Algebraic Geometry 2

Exercises 12

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Exercise 1. Let X be a scheme.

(1) Let $j : U \to X$ be an open immersion. Show that $j^*(-) \simeq (-)|_U$. Deduce that this is an exact functor.
(2) Let $i : Z \to X$ be a closed immersion. Show that in general, $i^* : \mathcal{O}_X\text{-Mod} \to \mathcal{O}_Z\text{-Mod}$ is *not* exact.

Exercise 2. Let X be a topological space. Establish the following.

(1) If $0 \to F_1 \to F_2 \to F_3 \to 0$ is an exact sequence of abelian sheaves and F_1 is flasque, then $0 \to F_1(X) \to F_2(X) \to F_3(X) \to 0$ is exact.
(2) If $0 \to F_1 \to F_2 \to F_3 \to 0$ is an exact sequence of abelian sheaves and F_1, F_2 are flasque, then also F_3 is flasque.
(3) If $f : X \to Y$ is a continuous map and F is flasque on X, then f_*F is flasque on Y.

Exercise 3. Let $\mathcal{E}$ be a locally free sheaf of rank n on the scheme X. Put $V(\mathcal{E}) := \mathrm{Spec}\,(Sym(\mathcal{E}))$ and write $p : V(\mathcal{E}) \to X$ for the canonical projection.

(1) Show that for $x \in X$ there exists an open neighbourhood U such that $p^{-1}(U) \simeq \mathbb{A}^n_U$ as U-schemes.
(2) Show that the sheaf of sections of $V(\mathcal{E})$ is $\mathcal{E}^\vee$.

Exercise 4. Let I be an injective abelian sheaf on the topological space X, and $U \subset X$ an open subspace. Show that $I|_U$ is injective on U. Deduce that if $f : X \to Y$ is a continuous map and F is an abelian sheaf on X, then $(R^i f_* F)(V)$ is the sheaf associated with the presheaf

$$Y \supset V \mapsto H^i(f^{-1}(V), F).$$