Algebraic Geometry 2

Exercises 11

Dr. Tom Bachmann

Exercise 1. Let $\varphi : A \to B$ be a homomorphism of commutative rings. For a B-module N, denote by N^φ the induced A-module. Show that the functor

$$B\text{-Mod} \to A\text{-Mod}, \quad N \mapsto N^\varphi$$

is right adjoint to $M \mapsto B \otimes_A M$.

Exercise 2. Let $\varphi : A \to B$ be a homomorphism of commutative rings and write $f : \operatorname{Spec}(B) \to \operatorname{Spec}(A)$ for the corresponding morphism of affine schemes. For a B-module N, show that

$$f_* \widetilde{N} \simeq \widetilde{N^\varphi}.$$

Exercise 3. Let A be a UFD and Σ a locally free sheaf of rank 1 on $\operatorname{Spec}(A)$. Show that Σ is trivial (i.e. isomorphic as an $\mathcal{O}_{\operatorname{Spec}(A)}$-module to $\mathcal{O}_{\operatorname{Spec}(A)}$).

Exercise 4. Let Σ be a locally free sheaf of finite type on the scheme X.
  (1) Show that the canonical map $\Sigma \to (\Sigma^\vee)^\vee$ is an isomorphism.
  (2) Construct for any $\mathcal{O}_X$-module M a canonical isomorphism

$$\mathcal{H}om_{\mathcal{O}_X}(\Sigma, M) \simeq \Sigma^\vee \otimes M.$$