Algebraic Geometry 2

Exercises 1

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Exercise 1. Let $X \to Y$, $X' \to Y'$ be closed immersions of S-schemes. Show that $X \times_S X' \to Y \times_S Y'$ is also a closed immersion.

Exercise 2. Consider morphisms of schemes

$$X \xrightarrow{f} Y \xrightarrow{g} S$$

Suppose that g is separated and $g \circ f$ is affine. Show that f is affine.

Exercise 3. Let R be a valuation ring with fraction field K and X a scheme. Put $T = \operatorname{Spec} R$. Recall that T has a closed point y_0 and an open point y_1. Show that the assignment

$$(f : T \to X) \mapsto (f(y_0), f(y_1), f^\sharp : \kappa(f(y_1)) \to K)$$

induces a bijection between $\operatorname{Hom}(T, X)$ and the set of triples (x_0, x_1, α) where $x_1 \in X$, $\alpha : \kappa(x_1) \to K$, $x_0 \in \overline{\{x_1\}} =: Z$, and R dominates $\mathcal{O}_{Z,x_0}$ (where Z is given the reduced closed subscheme structure).

Exercise 4. Let $f : X \to Y$ be a morphism of schemes. Show that f is separated if and only if for some open covering $\{U_i \subset Y\}_i$ the base changes $f_i : X \times_Y U_i \to U_i$ are separated.