

Algebraic Geometry 2

Exercises Tutorium 8

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Exercise 1. Recall the meaning of a rational curve over k . Classify non-singular rational curves over k .

Exercise 2. Let k be a field of characteristic $\neq 2$ and denote by Y the affine curve $V(y^2 - x^3 + x) \subset \mathbb{A}_k^2$. Prove that Y is not rational, as follows.

- (1) Show that Y is non-singular.
- (2) Show that $k[Y]$ is a domain.*
- (3) Show that the subring of $k[Y]$ generated by x over k is isomorphic to the polynomial ring $k[x]$.
- (4) Construct an automorphism σ of $k[Y]$ with $\sigma(x) = x$ and $\sigma(y) = -y$.
- (5) For $a \in k[Y]$ define $N(a) = a \cdot \sigma(a)$. Show that $N(a) \in k[x]$, $N(1) = 1$ and $N(ab) = N(a)N(b)$.
- (6) Show that $k[Y]^\times = k^\times$.
- (7) Show that $x, y \in k[Y]$ are irreducible.
- (8) Show that $k[Y]$ is not a UFD.
- (9) Conclude.

*Note: this is not really necessary. Why?