Algebraic Geometry 2

Exercises Tutorium 6

Dr. Tom Bachmann Winter Semester 2020–21

Exercise 1. Let X and Z be integral schemes of finite type over a field k, with function fields K and F. Let L/K be a finite field extension and denote by $Y \to X$ the normalization of X in L. Assume that Z is normal.

Show that a dominant morphism $Z \to Y$ is the same as a pair of a dominant morphism $Z \to X$ and an embedding $L \hookrightarrow F$ extending $K \hookrightarrow F$.

[*Hint:* reformulate as a lifting problem and use Exercise 1 of this week's ZA.]

Exercise 2. Let $\{A_i\}_{i \in I}$ be integrally closed subrings of the field K, each with field of fractions K. Show that $A := \bigcap_i A_i$ is integrally closed. Construct an example where $Frac(A) \neq K$.

Exercise 3. Let $A \subset B$ be dvrs with $Frac(A) = Frac(B)$. Show that $A = B$.