Algebraic Geometry 2

Exercises Tutorium 12

Dr. Tom Bachmann — Winter Semester 2020–21

Exercise 1. Let $S_\bullet$ be a graded ring and $M_\bullet$ a graded module. Construct a quasi-coherent sheaf $\widetilde{M_\bullet}$ on $\mathbf{Proj}(S_\bullet)$ such that $\widetilde{M_\bullet}|_{D_+(f)} \simeq \widetilde{M_{(f)}}$.

Exercise 2.

(1) Let X be a scheme, $\mathcal{L}$ an invertible sheaf on X and $\alpha : \mathcal{O}_X^{n+1} \to \mathcal{L}$ be a surjection. Let $U_i \subset X$ be the open subset where the section $\alpha(e_i)$ of $\mathcal{L}$ does not vanish (make precise what this means!). Here e_i is the canonical basis section of $\mathcal{O}_X^{n+1}$. Show that the U_i form an open cover of X, and that we have canonical isomorphisms $\mathcal{L}|_{U_i} \simeq \mathcal{O}_X$.

(2) On $\mathbb{P}^n_{\mathbb{Z}}$ consider the sheaf $\mathcal{O}(1) := \widetilde{M_\bullet}$, where $M_i = \mathbb{Z}[T_0, \dots, T_n]_{i+1}$. Show that $\mathcal{O}(1)$ is invertible and construct a surjection $\mathcal{O}^{n+1} \to \mathcal{O}(1)$.

(3) Consider the functor $F : Sch^{op} \to Set$ such that $F(X)$ is the set of invertible sheaves on X together with a surjection from $\mathcal{O}_X^{n+1}$ (make this precise!). Show that F is isomorphic to the functor represented by $\mathbb{P}^n_{\mathbb{Z}}$.