Algebraic Geometry 2

Exercises Tutorium 11

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Let $f : X \to Y$ be a morphism of schemes.

Exercise 1. Show that the functor $f_* : \mathcal{O}_X\text{-Mod} \to \mathcal{O}_Y\text{-Mod}$ admits a left adjoint, namely $f^*(-) := f^{-1}(-) \otimes_{f^{-1}\mathcal{O}_Y} \mathcal{O}_X$.

Exercise 2. Show that the functor f^* from (1) is symmetric monoidal, i.e.

$$f^*(M) \otimes_{\mathcal{O}_X} f^*(N) \simeq f^*(M \otimes_{\mathcal{O}_Y} N).$$

Exercise 3.

(1) For M an $\mathcal{O}_X$-module and N an $\mathcal{O}_Y$-module, construct a natural map

$$f_*(M) \otimes_{\mathcal{O}_X} N \to f_*(M \otimes_{\mathcal{O}_X} f^*N).$$

(2) If N is locally free (of finite rank), show that the map is an isomorphism.