Algebraic Geometry 2

Exercises Tutorium 10

Dr. Tom Bachmann Winter Semester 2020–21

Exercise 1. Let X be an affine scheme and $0 \to \mathcal{F}_1 \to \mathcal{F}_2 \to \mathcal{F}_3 \to 0$ be an exact sequence of $\mathcal{O}_X$-modules. Show that if $\mathcal{F}_1$ is quasi-coherent then $0 \to \mathcal{F}_1(X) \to \mathcal{F}_2(X) \to \mathcal{F}_3(X) \to 0$ is exact.

[*Hint*: First show that given $a \in \mathcal{F}_3(X)$ lifting to $\mathcal{F}_2(D(f))$, for n sufficiently large there exists a lift of $f^n a$ to $\mathcal{F}_2(X)$.]

Exercise 2. Let X be a scheme. Show that the kernel, cokernel, and image of any morphism of quasi-coherent sheaves is quasi-coherent. Show that if $0 \to \mathcal{F}_1 \to \mathcal{F}_2 \to \mathcal{F}_3 \to 0$ is an exact sequence of $\mathcal{O}_X$-modules with $\mathcal{F}_1, \mathcal{F}_3$ quasi-coherent, then $\mathcal{F}_2$ is quasi-coherent.