Algebraic Geometry 2

Exercises Tutorium 1

Dr. Tom Bachmann — Winter Semester 2020–21

Exercise 1. Let S be a scheme, X a reduced S-scheme and Y a separated S-scheme. Suppose given S-morphisms $f, g : X \to Y$ agreeing on a dense open subset of X. Show that $f = g$.

Show by example that neither the assumption on X nor on Y can be removed.

Exercise 2. Let X be a separated scheme. Show that affine open subsets of X are stable under binary intersection.

Exercise 3. Let X be a separated integral scheme of finite type over a field k, with function field K. Show that if R is a valuation ring of K, then there exists at most one point $x \in X$ with $\mathcal{O}_{X,x} \subset R$ and $m_x \subset m_R$.

Exercise 4. Show that valuation rings are integrally closed.

Exercise 5 (Extra problem)**.** Show that an integral domain R is a valuation ring if and only if given ideals I, J we have either $I \subset J$ or $J \subset I$. Deduce that if R is a valuation ring and P is a prime ideal, then R/P and R_P are valuation rings.