Algebraic Geometry 1

Exercises 5

Dr. Tom Bachmann Winter Semester 2020–21

Exercise 1. Let X be a topological space and S a set, viewed as a topological space with discrete topology. Denote by $\underset{\sim}{S}$ the constant *presheaf* with value S, and by $\underline{S}$ the constant *sheaf*

$$\underline{S} = C(-, S),$$

where $C(-,-)$ denotes the set of continuous functions. Show that the canonical morphism of presheaves $\underset{\sim}{S} \to \underline{S}$ induces an isomorphism on stalks at all points of X, but is not an isomorphism of presheaves unless $|S| = 1$.

Exercise 2. Let X be an affine variety over the algebraically closed field k. Denote by

$$\mathcal{O} : U \mapsto \mathcal{O}(U)$$

the sheaf of regular functions on X. Let $x \in X$ and $m_x \subset \mathcal{O}(X)$ the corresponding maximal ideal. Show that

$$\mathcal{O}_x \simeq \mathcal{O}(X)_{m_x}.$$

(Here the left hand side denotes the stalk of a sheaf and the right hand side the localization of a ring.)

Exercise 3. Let F be a sheaf of abelian groups on a topological space X and $s \in F(X)$. Show that

$$sup(s) := \{x \in X \mid s_x \neq 0 \in F_x\} \subset X$$

is closed.

Exercise 4. Let F, G be presheaves on a topological space X.

(1) Show that for any $x \in X$ we have

$$(F \times G)_x \simeq F_x \times G_x.$$

(2) Show that if F, G are sheaves then so is $F \times G$.