Algebraic Geometry 1

Exercises 4

Dr. Tom Bachmann Winter Semester 2020–21

Throughout let k be an algebraically closed field.

Exercise 1. Let A be an integral domain with field of fractions K. Show that

$$A = \bigcap_m A_m \subset K,$$

where the intersection is over all maximal ideals m of A.

Exercise 2. Show that

$$\phi : \mathbb{A}^1_k \to Z(X^3 - Y^2) \subset \mathbb{A}^2_k, T \mapsto (T^2, T^3)$$

defines a morphism of varieties. Is ϕ an isomorphism?

Exercise 3. Show that

$$\phi : Z(XY - 1) \subset \mathbb{A}^2_k \to \mathbb{A}^1_k \setminus 0, (X, Y) \mapsto X$$

defines a morphism of varieties. Is ϕ an isomorphism?

Exercise 4. Let X be a quasi-projective variety over k. Show that there exists an open covering of X by *affine* varieties.