Algebraic Geometry 1

Exercises 2

Dr. Tom Bachmann Winter Semester 2020–21

Throughout let k be an algebraically closed field.

Exercise 1. Show that the map

$$\mathbb{A}^1(k) \to \mathbb{A}^2(k), T \mapsto (T^2, T^3)$$

is a homeomorphism onto

$$Z(X^3 - Y^2) \subset \mathbb{A}^2(k).$$

Here T denotes the coordinate on $\mathbb{A}^1$, and X, Y the coordinates on $\mathbb{A}^2$.

Exercise 2. Let

$$Z = Z(XY - 1) \subset \mathbb{A}^2(k).$$

Show that Z is irreducible and the projection

$$\mathbb{A}^2(k) \to \mathbb{A}^1(k), (X, Y) \mapsto X$$

induces a homeomorphism between Z and $\mathbb{A}^1(k) \setminus \{0\}$.

Exercise 3. Show that the map

$$\mathbb{A}^1(k) \to \mathbb{A}^3(k), T \mapsto (T^3, T^4, T^5)$$

is a homeomorphism onto $Z(P) \subset \mathbb{A}^3(k)$, for some prime ideal P of $k[X, Y, Z]$. Show that $ht(P) = 2$.

Exercise 4. Let

$$\pi : \mathbb{A}^{n+1}(k) \setminus \{0\} \to \mathbb{P}^n(k)$$

be the canonical epimorphism, $Z \subset \mathbb{P}^n(k)$ closed and

$$\pi^{-1}(Z) = \bigcup_i Y_i$$

the decomposition of $\pi^{-1}(Z)$ into irreducible components. Show that each Y_i is $k^\times$-invariant.