Algebraic Geometry 1

Exercises 1

Dr. Tom Bachmann Winter Semester 2020–21

Exercise 1. Let R be a (commutative, unital) ring and I, J ideals in R. Show that

$$\sqrt{IJ} = \sqrt{I \cap J} = \sqrt{I} \cap \sqrt{J} \supset I \cap J \supset IJ.$$

Exercise 2. Let k be an infinite field and $f \in k[X_1, \ldots, X_n]$ be non-constant. Show that there exists $x \in k^n$ such that $f(x) \neq 0$.

Exercise 3. Let X be a topological space.

(1) Let X be irreducible and U a non-empty open subset of X. Show that U is irreducible and $\overline{U} = X$.

(2) Let Y be a subset of X. Show that Y is irreducible if and only if $\overline{Y}$ is irredcible.

Exercise 4. Let k be an algebraically closed field.

(1) Let $Y \subset \mathbb{A}^n(k)$ be closed. Establish a bijection between closed subsets of Y and radical ideals in $k[Y] := k[X_1, \ldots, X_n]/I(Y)$.

(2) Show that if A is a finite type, reduced k-algebra, then there exist an algebraic set Y over k and an isomorphism of k-algebras $k[Y] \simeq A$.