Algebraic Geometry 1

Exercises Tutorium 9

Dr. Tom Bachmann Winter Semester 2020–21

Exercise 1. Let $f : X \to Y$ be a morphism of schemes and $U \subset X$, $V \subset Y$ open subschemes such that $f(U) \subset V$. Show that there is a unique morphism of schemes $f' : U \to V$ such that the following diagram commutes

$$\begin{tikzcd} U \arrow[r] \arrow[d, "f'"'] & X \arrow[d, "f"'] \\ V \arrow[r] & Y. \end{tikzcd}$$

Exercise 2. Let $f : X \to Y$ be a morphism of schemes. Show that f is an open immersion of schemes (i.e. $|f| : |X| \to |Y|$ is a homeomorphism onto an open subset and $f^{\#}$ induces an isomorphism on stalks) if and only if f is an isomorphism onto an open subscheme.

Exercise 3. Call a morphism $f : X \to Y$ of schemes *quasi-compact* if there exists an affine open cover $\{V_i\}_i$ of Y such that $f^{-1}(V_i)$ is quasi-compact for every i. Show that f is quasi-compact if and only if for every affine open subset V of Y, $f^{-1}(V)$ is quasi-compact.

Exercise 4. Let X be a reduced scheme. Given $Z \subset X$ closed, denote by $I_Z \subset \mathcal{O}_X$ the subsheaf consisting of those regular functions vanishing along Z (in the sense of Ex. 4 sheet 8). Show that this induces an injection from the set of closed subsets of X to the set of subsheaves of $\mathcal{O}_X$. Can you characterize its image?