

Algebraic Geometry 1

Exercises Tutorium 7

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Exercise 1. Give an example of a morphism of sheaves which is an epimorphism, but not an epimorphism when viewed as a morphism of presheaves.

Exercise 2. Give an example showing that the forgetful functor from sheaves to presheaves does not preserve binary coproducts (in general).

Exercise 3. Let X be an affine algebraic variety over the algebraically closed field k . Show that $\mathcal{O}_X(X)$ can be identified with the set of morphisms from X to \mathbb{A}_k^1 .

Exercise 4. Show that in the previous exercise, the assumption “affine” may be omitted.

Exercise 5. Let X be an algebraic variety over the algebraically closed field k . Given $Z \subset X$ closed, denote by $I_Z \subset \mathcal{O}_X$ the subsheaf consisting of those regular functions vanishing along Z . Show that this induces an injection from the set of closed subsets of X to the set of subsheaves of \mathcal{O}_X . Can you characterize its image?